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Self-dual nonsupersymmetric type II string compactifications

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Abstract: It has recently been proposed that certain nonsupersymmetric type II orbifolds have vanishing perturbative contributions to the cosmological constant. We show that techniques of Sen and Vafa allow one to construct dual type II descriptions of these models (some of which have no weakly coupled heterotic dual). The dual type II models are given by the same orbifolds with the string coupling S and a T^2 volume T exchanged. This allows us to argue that in various strongly coupled limits of the original type II models, there are weakly coupled duals which exhibit the same perturbative cancellations as the original models.

Keywords: Superstring Vacua, String Duality, Supersymmetry Breaking.

Contents

1. Introduction

Motivated in part by the AdS/CFT correspondence [1, 2], it was recently suggested that certain special nonsupersymmetric type II string compactifications might nevertheless have vanishing cosmological constant Λ [3]. A concrete candidate example (with supporting perturbative computations at the one and two loop level, as well as a more heuristic argument for higher-loop cancellations using the general form of the higher genus twist structures) was presented in [4].[1]

Unfortunately, explicit checks at higher loops are generally difficult to perform, and provide no insight into possible nonperturbative corrections to the vacuum energy. In a recent paper [5], Harvey pointed out that techniques from string duality can be fruitfully applied to models like the one studied in [4] (for some discussions of duality and nonsupersymmetric string vacua in other contexts see e.g. [10]). By computing at one loop in a dual heterotic model, he finds that at a given order in a type II α' expansion, perturbative contributions in the type II coupling cancel to all orders, providing a test of the all-orders vanishing conjectured in [4]. Moreover, he finds nonzero non-perturbative contributions. The precise model studied in [5] is a slight variant of the model proposed in [4]; the difference between the models is crucial in enabling one to construct a heterotic dual for the modified model.

[1] Subtleties in the gauge choice at two loops in [4] are being investigated. At higher loops there is no known consistent method for fixing the gravitino gauge slice in superstring perturbation theory, so we are left so far with a more heuristic understanding of these contributions and how they appear to cancel.

In this note, we point out that both models of the type studied in [4] and the modified version discussed in [5] (as well as some new models which share their interesting features) can also be given dual type II descriptions in certain limits. In fact, these models are very special – they turn out to be $self-dual$ as type II vacua. By this we mean that the models with given values of the coupling S and a T^2 volume modulus T are dual to the same orbifolds, with S and T interchanged. This allows us to argue that in various strongly coupled regimes of the original theory, there exists a dual weakly coupled type II theory which also has vanishing leading perturbative contributions to the cosmological constant. Moreover, in the specific model of [4] as well as in the new models to be introduced here, there is no weakly coupled heterotic limit and all type II duals enjoy the same perturbative cancellations that occur in the original model. This does not imply that Λ vanishes exactly in the models without heterotic duals, but it does mean that any concrete checks we can perform using string duality (along the lines of [5]) are consistent with such vanishing.

In section 2, we briefly review the construction [6] which allows us to systematically determine the type II duals. In section 3, we apply this construction to the models of interest and find that they are self-dual. The implications of this finding for the models which do and do not admit heterotic duals are described. We close with a brief discussion in section 4.

2. Brief review of type II orbifold dual pairs

2.1. Basic idea

A systematic construction of dual pairs of type II compactifications in $D = 4, 5$ dimensions was discussed by Sen and Vafa [6]. The U-duality group of type II compactifications on T^4 is $SO(5,5;\mathbb{Z})$, while the perturbatively obvious (T-duality) subgroup is $SO(4,4;\mathbb{Z})$. Consider two elements $h, \tilde{h} \in SO(4,4;\mathbb{Z})$ of order n, which are not conjugate in $SO(4,4;\mathbb{Z})$ but which *are* conjugate in $SO(5,5;\mathbb{Z})$:

$$ghg^{-1} = \tilde{h}\,, \qquad g \in SO(5,5;\mathbb{Z})\,. \tag{2.1}$$

There is a subspace $\mathcal{M}$ of the moduli space of T^4 compactifications which consists of theories with an extra h symmetry (which one finds by looking for fixed points of the h action on the Teichmüller space). Using the U-duality transformation g, this is dual to the subspace $\tilde{\mathcal{M}}$ invariant under $\tilde{h}$. Since g is not in $SO(4,4;\mathbb{Z})$, the duality is not obvious perturbatively. On $\mathcal{M}$ and $\tilde{\mathcal{M}}$, h and $\tilde{h}$ are realized as symmetries of the compactifications in question.

Now, consider compactifying on an additional T^2, which we can take to be a product of two circles. We can orbifold the resulting compactifications by h ($\tilde{h}$) acting on the T^4 combined with a free order n action on the T^2. By the adiabatic argument [7], the resulting models in four dimensions are still dual.

2.2. Element of interest

The particular $SO(5,5;\mathbb{Z})$ element $\bar{g}$ of interest is given in terms of the element σ of the ten-dimensional $SL(2,\mathbb{Z})$ symmetry group of type IIB which inverts the ten-dimensional axion/dilaton field and the T-duality element τ_{1234} that inverts the volume of the T^4:

$$\bar{g} = \sigma \cdot \tau_{1234} \cdot \sigma^{-1}. \tag{2.2}$$

This maps fundamental strings (without winding on the T^4) to NS fivebranes wrapped on the T^4. The element $\bar{g}$ has the very helpful property that

$$\bar{g} h \bar{g}^{-1} \in SO(4,4;\mathbb{Z}). \tag{2.3}$$

for all $h \in SO(4,4;\mathbb{Z})$ [6].

One interesting and important property of the element $\bar{g}$ is that, as long as the Ramond-Ramond scalar VEVs in $D = 6$ vanish, it acts on the dilaton and the three-form field $H_{\mu\nu\rho}$ in exactly the same way as the type IIA / heterotic string-string duality. Therefore, after the further compactification on an (orbifolded) T^2, the dual models that the adiabatic argument yields will be related by $S - T$ exchange:

$$\tilde{S} = T, \qquad \tilde{T} = S, \tag{2.4}$$

where T is the Kähler modulus associated with the T^2 and S is the axion-dilaton in four dimensions. This map is inherited from the supersymmetric theory where certain quantities are holomorphic in S and T. In the nonsupersymmetric theory nothing is determined by holomorphic objects, and generic computations will depend on both S and $S^\dagger$ as well as T and $T^\dagger$.

By detailed considerations we will not repeat here, Sen and Vafa derive the result of conjugating various elements of $SO(4,4;\mathbb{Z})$ by $\bar{g}$. Consider an element $h \in SO(4,4;\mathbb{Z})$ that acts on the 8-vector $X_L^{1\ldots4}, X_R^{1\ldots4}$ as

$$h = \begin{pmatrix} \omega(\theta_L) & & & \\ & \omega(\phi_L) & & \\ & & \omega(\theta_R) & \\ & & & \omega(\phi_R) \end{pmatrix}, \tag{2.5}$$

where

$$\omega(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \tag{2.6}$$

We will abuse notation and denote elements of $SO(4,4;\mathbb{Z})$ which are like (2.5) by $(\theta_L, \phi_L, \theta_R, \phi_R)$. Then $\bar{g}$ conjugates h to $\tilde{h}$ which acts on $X_L^{1\ldots4}, X_R^{1\ldots4}$ as $(\tilde{\theta}_L, \tilde{\phi}_L, \tilde{\theta}_R, \tilde{\phi}_R)$, where

$$\begin{pmatrix} \tilde{\theta}_L \\ \tilde{\phi}_L \\ \tilde{\theta}_R \\ \tilde{\phi}_R \end{pmatrix} = \begin{pmatrix} 1/2 & -1/2 & 1/2 & -1/2 \\ -1/2 & 1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 \\ -1/2 & -1/2 & 1/2 & 1/2 \end{pmatrix} \begin{pmatrix} \theta_L \\ \phi_L \\ \theta_R \\ \phi_R \end{pmatrix} \tag{2.7}$$

(2.7) is the equation that will yield the type II duals of our orbifolds. We will also discuss directly how the map arises in our particular examples.

3. Application to nonsupersymmetric models

3.1. Original model

The model of [4] is constructed by orbifolding type II on a T^6 consisting of a product of six circles. The orbifold group has two generators f and g. In table 1 we have indicated how each element acts on the left and right moving RNS degrees of freedom of the superstring. s refers to a shift by $R/2 = l_s/2\sqrt{2}$.

S^1	f	g
1	$(-1, s)$	$(s, -1)$
2	$(-1, s)$	$(s, -1)$
3	$(-1, s)$	$(s, -1)$
4	$(-1, s)$	$(s, -1)$
5	$(s^2, 0)$	(s, s)
6	(s, s)	$(0, s^2)$
	$(-1)^{F_R}$	$(-1)^{F_L}$

Table 1: Action of f, g.

If we concentrate on the action on the first four circles, then we see that in the notation of subsection 2.2 f and g can be represented as

$$f = (\pi, \pi, 2\pi, 0), \qquad g = (2\pi, 0, \pi, -\pi)\,, \tag{3.1}$$

(where we use the fact that e.g. $(-1)^{F_R}$ can be represented by a 2π rotation on right movers). Then from the action of (2.7), we see that $\tilde{f} = f$ and $\tilde{g} = g$. So after composing with the further action on the $X^{5,6}$ T^2, we will find that the f orbifold is self-dual as is the g orbifold, and so is the orbifold by both f and g.

This self-duality can be understood in terms of the duality between the fundamental string and the wrapped NS fivebrane mentioned above. As explained in [9], there are normalizable RR zero modes of the NS fivebrane associated to the 2-forms on the T^4. These zero modes correspond to the worldsheet embedding coordinates in the dual type II string. The element f in the original model kills half the RR fields. This means that the dual element $\tilde{f}$ must act with a (-1) on half the 8 worldsheet scalars $\tilde{X}^i_L, \tilde{X}^i_R, i = 1, \ldots, 4$ associated with the dual T^4. It also preserves $1/4$ of the supersymmetry. Since it preserves some supersymmetry, the dual element $\tilde{f}$ must act with a (-1) on only the left-movers or only the right-movers. To kill $3/4$ of the supersymmetry it must also involve an action $(-1)^{F_R}$ or $(-1)^{F_L}$ respectively. Thus $\tilde{f}$ is isomorphic to f. The element $\tilde{g}$ has similar properties, but kills the rest of the supersymmetry, so it must be isomorphic to g.

We have not yet discussed the duals of the shifts involved in our group elements. As in [5], we must choose them to level-match. These shifts map to gauge transformations of the RR fields in the NS fivebrane background (as was first discussed in [8]). Such gauge transformations only have a non-perturbative effect on the dual side.

Now (2.4) tells us that we can do some strong coupling computations in the original theory by doing small radius (of the T^2) computations in the dual. On the other hand, the dual coupling is related to the Kähler modulus T of the T^2 in the original theory. Therefore, one can analyze the theory in several different regimes:
1) If the original theory is weakly coupled ($S \to \infty$) and at arbitrary T, then the analysis of [4] goes through and at least the leading perturbative contributions to Λ as a function of S vanish.

2) Next let us look at the dual model in its perturbative regime, i.e. $\tilde{S} \to \infty$ and arbitrary $\tilde{T}$. As just discussed, this model is isomorphic to the original model. Therefore in this regime, perturbatively (at least to two loops) in $\tilde{S} = T$ and to all orders and non-perturbatively in $\tilde{T} = S$ (at these orders in $\tilde{S}$), we find no contribution. This is in contrast to the situation in [5], where a heterotic dual was obtained in this limit which also had no contributions perturbatively in $\tilde{T}$ but did have nonzero contributions non-perturbatively. In both cases one finds a test of the proposal that all perturbative contributions in $\tilde{T} = S$ should cancel.
3) Finally, if the original model is strongly coupled and at small radius $(S, T \to 0)$, then we should first T-dualize to get to strong coupling at large radius $T \to \infty$. Then, we can use the duality (2.4) to get the dual model at weak coupling $\tilde{S} = \infty$ and small radius. Then, since the model is self-dual and we are at weak coupling, we are back in situation 1) and the analysis of [4] applies.

The upshot is that (2.4) implies that in all these limits in $S - T$ space, there is a weakly coupled dual. Because the model is actually self-dual, the perturbative evaluation of Λ in the dual vanishes at the leading orders of perturbation theory exactly as in [4].

3.2. Some new models

In [4] we emphasized the non-abelian nature of the orbifold as a simple way to ensure that the 1-loop vacuum energy would cancel. In fact, as noted also in [5], this was not actually necessary in these models. Nonzero contributions could only occur in contributions to the 1-loop (torus) amplitude which involve twisting by group elements on the (a, b) cycles of the torus which break all of the supersymmetry. With the non-abelian structure, these never occur. If we remove the asymmetric shifts along the T^4 (and adjust the asymmetric shift along the 5th circle so as to satisfy level-matching), then f and g would commute, so the (f, g) twist would need to be taken into account (along with various other supersymmetry breaking twists that can be obtained from (f, g) by modular transformations, like (f, fg) and (g, fg)). This twist structure describes a trace over f-twisted states with an insertion of the g operator. It is simplest to consider the spectrum in light-cone gauge. In the f-twisted sector, the vacuum after GSO projection can easily be seen to have equal numbers of g-invariant and g-anti-invariant states. The right-moving Ramond vacuum is a spinor, and the right-moving NS vacuum is of the form $\psi^{1,\ldots,8}_{-1/2}|-1/2\rangle$. The four reflections in g kill half of each of these sets of states. This ensures that all mass levels have this property, and the contribution cancels, though the diagram has information about the full supersymmetry-breaking in the model. Similar remarks apply of course to the other supersymmetry breaking twists at 1-loop, which can be obtained from this by modular transformations.

This said, we now can consider another model where we remove the reflections in g and change the shifts to a single symmetric shift on X^6. Then again in the g-twisted sector, the f operator has equal numbers of f-invariant and f-anti-invariant states.

So the (g, f) (and other modular equivalent) supersymmetry breaking twist structure contributions vanish. One can also use U-duality to find type II duals which give one control over various strong-coupling limits of these models, as in subsection 3.1.

3.3. Models with heterotic duals

Finally we can study various limits of the model discussed in [5]. This model has five noncompact dimensions, arising from a compactification on the orbifold $(T^4 \times S^1)/\{f', g'\}$. The element f' differs from f above in that there is no shift on the sixth circle. The element g' differs from g in that its asymmetric shift $(0, s^2)$ acts on the fifth circle and there is no extra symmetric shift. This means that the element $f'g'$ (combined with a full lattice shift) creates a K3. The T^4 is square and at the self-dual radii while the S^1 has a variable radius R. We will also keep track of the six-dimensional string coupling λ. If we start with the type IIA string, in the limit $(\lambda_A \to \infty, R_A \to \infty)$ there is a weakly-coupled heterotic dual description [5]. Let us consider now the limit $\lambda_A \to \infty, R_A \to 0$. We can T-dualize this to type IIB in the limit $\lambda_B \to \infty, R_B \to \infty$ where the subscript B denotes the corresponding quantities $R_B = \alpha'/R_A$ and $\lambda_B = \lambda_A\sqrt{\alpha'}/R_A$ in the IIB theory. The IIB theory on K3 has the 10d S-duality symmetry σ as well as the volume inversion symmetry τ_{1234} that enter in $\bar{g}$ (2.2). This means that we can use this element to generate a type II/type II string-string dual in this regime for this model. Perturbative contributions to the cosmological constant in this regime will cancel as in the original model.

If duality is a valid technique for studying these models, the heterotic limit [5] implies that there is a nonvanishing dilaton potential (nonperturbative in the original type II coupling). As one shrinks R_A (moving from the regime of validity of the heterotic dual towards the regime of validity of the type II dual), the heterotic dual develops a tachyon. It is therefore not completely clear that the heterotic and type II duals are connected by changing parameters in the nonperturbative string model. In any case, the results of [5] seem to imply that in cases with heterotic duals the type II duals must also have a nontrivial dilaton potential of a form which is not ruled out by existing calculations.[2] It is clear however that the models of subsections 3.1 and 3.2 do not have a non-perturbative potential at the same order in the original type II α' and g_{st} as the contribution detected in [5].

4. Discussion

As we have seen, varying the way the shifts (and to some extent the reflections) occur in these orbifold models changes some of their non-perturbative properties significantly. In particular, some models have heterotic duals, while others only have type II duals. The trick used in [4] for cancelling perturbative contributions to the cosmological constant

[2] Very naive extrapolation of the results of [5] indicate that the dilaton potential would behave like $e^{-1/\tilde{\lambda}_B}$ in the dual type IIB theory, because it behaves like e^{-1/R_h} at small heterotic radius R_h.

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was specific to type II in that the freedom to use both $(-1)^{F_L}$ and $(-1)^{F_R}$ was crucial. It is therefore reassuring that some models only have type II duals from the point of view of aiming for non-perturbative cancellation (although our results do not imply that there is exact cancellation in any of the models discussed here, since duality considerations only allowed us to check for a small subset of the possible nonperturbative contributions). An exactly flat dilaton potential in similar 3d models could be of interest for potentially generating a four-dimensional model with no cosmological constant upon taking the strong-coupling limit. On the other hand, if one considers this type of model in 4d, one might ultimately want models with non-perturbative dilaton potentials [5], either to stabilize the dilaton or to agree with observations suggesting a small nonzero vacuum energy. Of course we are still far from a truly realistic model in which to usefully discuss these issues.

It is interesting that these slight variations in the shifts used change the dual descriptions so drastically. As mentioned in [5], it would be very interesting to study the D-brane spectra in these models in order to understand their degeneracy (or lack thereof) in the various orbifold formulations.

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